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COMMENT

Position-space renormalisation group for SAWs on the L lattice

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Abstract. The self-avoiding walk problem on the L lattice is treated with a position-space renormalisation group approach using a 'centre-rule' weight function.

An L lattice is a directed square lattice having the property that each bond on a path must be directed at right angles to its predecessor. Such a lattice has also been referred to as the two-choice 90° lattice (Wall *et al* 1955) and the underlying Manhattan square lattice (Kasteleyn 1963, Malakis 1975, 1984). The name L lattice is due to the fact that a self-avoiding walk (SAW) on such a lattice can be viewed as a concatenation of an L-shaped, two-step walk (Guttmann 1983).

The purpose of this comment is to try to improve the position-space renormalisation group (PSRG) approach of Malakis (1984) for the L lattice using the approach proposed by Chao (1985) for SAWs on an ordinary lattice. Although the results of Malakis (1984) seem to be reasonable and are able to demonstrate that SAWs on the L lattice belong to the same universality class as those on the non-oriented lattice, there are some inconsistencies in the renormalisation scheme that will be pointed out in our discussion.

The renormalisation scheme of Chao (1985) for a SAW on a square lattice adapts a 'centre-rule' weight function, which is a modified version of the 'corner rule' weight function of de Queiroz and Chaves (1980) and the similar approaches of Redner and Reynolds (1981). Basically, it is a straightforward extension of the scheme commonly used for a PSRG treatment for bond percolation on the square lattice (Reynolds *et al* 1977). Therefore, for SAWs on the L lattice, we will use the family of cells illustrated in figure 1. Without bond orientation, such cells are exactly the same as those used

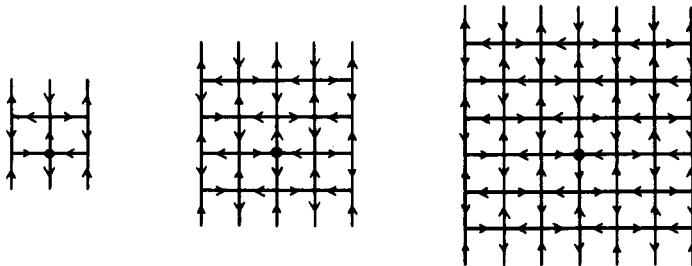


Figure 1. Cells with size $b = 3, 5$ and 7 used for the PSRG treatment for the L lattice.

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for SAWs on the ordinary square lattice. It should be noted that other than the origin for SAWs to start, such cells also have bonds on the bottom edges deleted in contrast with those of Malakis (1984).

Similar to the case of SAWs on the ordinary square lattice, the cells with size b even and odd should be grouped separately, due to differences in the location of the origin in the cells. We will treat only the b -odd cells which can have a majority of bonds pointing either upward or downward. The renormalisation of such cells should preserve the direction of the majority of bonds. The cells illustrated in figure 1 have a majority of bonds pointing upward. Also, for the b -odd cells, either one of the two sites in the middle of the central vertical line can be chosen as the origin. The origins of the cells shown in figure 1 are chosen consistently in the sense that all corresponding bonds incident from the origin of each cell have the same direction. We then sum the statistical weights for all spanning SAWs that traverse a cell of size b in the upward direction via the origin to obtain a polynomial G_b as a function of the fugacity K ; such polynomial is then renormalised to a bond directed upward having fugacity K' to yield a recursion relation

$$K' = G_b(K). \quad (1)$$

From such recursion relation, we obtain the non-trivial fixed point K_b^* and the critical exponent ν_b through the formula (see Stanley *et al* 1982)

$$\nu_b = \frac{\ln b}{\ln \lambda} \quad (2)$$

with $\lambda = (dG_b(K)/dK)|_{K^*}$.

Malakis (1984) has considered a cell-to-bond PSRG using a $b = 3$ cell with bottom edge included. The 'spanning' of the cell is defined as the average of the set of SAWs that begin at each of the three vertices at the bottom edge and exit by way of the top edge. Thus, both SAWs that begin with a vertical step as well as a horizontal step are considered. However, since all SAWs have to exit by way of a vertical step, they should always begin with a horizontal step on the bottom edge of a cell so as to be consistent with the characteristic of the walk. In our treatment, it should be understood that the interconnection of paths at the 'interfacing' between cells is always via the horizontal bonds omitted in the treatment. In our opinion, it is necessary to delete the bottom edge of the cells, so as to avoid inconsistency as well as ramified configurations in the 'interfacing'. Furthermore, Malakis has considered only the $b = 3$ case. Judging from the defects of his method, it is therefore questionable that the results using larger cells can converge to a correct value.

For the $b = 3$ cell, we readily obtain

$$G_3(K) = 4K^5 \quad (3)$$

which when renormalised to a bond gives the fixed point $K_3^* = 0.7071$ and $\nu_3 = 0.6826$.

For $b = 5$, the graph in figure 1 can be divided into four graphs illustrated in figure 2 according to the orientation of bonds at the origin. The spanning SAWs in figure 2 can be enumerated more easily and the sum gives

$$G_5(K) = 2K^{17} + 4K^{13} + 16K^9 \quad (4)$$

which yields $K_5^* = 0.7013$ and $\nu_5 = 0.7224$. Using a similar scheme, it is still possible

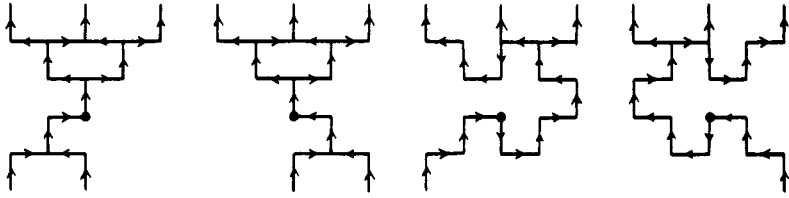


Figure 2. The $b = 5$ cell is divided into four graphs. Spanning SAWs in the upward direction are considered.

to enumerate all the SAW paths to obtain

$$G_7(K) = 2K^{37} + 40K^{33} + 70K^{29} + 88K^{25} + 100K^{21} + 88K^{17} + 64K^{13}. \tag{5}$$

The renormalisation of G_7 into a bond gives $K_7^* = 0.6874$ and $\nu_7 = 0.7279$. The results of cell-to-bond renormalisation indicate that K^* decreases whereas ν increases as the cell size b increases.

It is expected that the value of ν should converge to the exact one as $b \rightarrow \infty$. Such convergence behaviour can be seen from the graph for ν^{-1} versus $1/\ln b$ illustrated in figure 3. The extrapolation is by no means conclusive but indicates the possibility for approaching $\nu = 0.75$.

We have also calculated results for cell-to-cell renormalisation using equations similar to (1) and (2). The renormalisation of $b = 5$ cell to $b' = 3$ cell can be obtained by using the results of (3) and (4) to yield $K_{5/3}^* = 0.6960$ and $\nu_{5/3} = 0.8272$. Similarly, using (3), (4) and (5), we obtain $K_{7/3}^* = 0.6792$ and $\nu_{7/3} = 0.8007$ together with $K_{7/5}^* = 0.6656$ and $\nu_{7/5} = 0.7699$. Such results indicate a decreasing behaviour for both K^* and ν as b/b' decreases and that the results of ν may converge toward the same value as the case for cell-to-bond renormalisation.

Our results for cell-to-cell approach indicate tht the critical fugacity K_c should be less than 0.6656, which is consistent with that of Guttmann (1983).

In conclusion, we have treated the SAW problem on the directed L lattice with the same scheme as that for a non-directed lattice and obtained results in agreement with

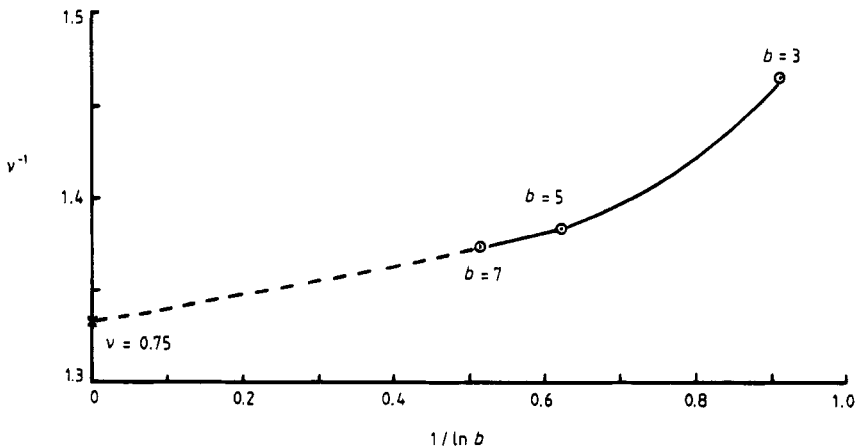


Figure 3. A plot for ν^{-1} as a function of $1/\ln b$ for cell-to-bond renormalisation with points corresponding to results for $b = 3, 5$ and 7 .

those of Guttman (1983) and supporting universality behaviour pointed out by Malakis (1975, 1984).

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